SIMILARITY SOLUTION FOR ONE-DIMENSIONAL STRONG EXPLOSION IN THE PERFECT GAS

K. L. Stepanov and S. M. Zen'kevich

Heat and Mass Transfer Institute, National Academy of Sciences of Belarus,
15, P.Brovka Street, 220072, Minsk, Belarus, e-mail: kls@hmti.ac.by

The strong explosion theory has arisen from the necessity to describe in the environment the distributions of shock waves caused by explosion of charges, having large specific energy both in small weight and in volume. This theory was developed in the works of L.I. Sedov, and also G. Taylor, K.P. Stanjukovich and J. Neumann. The analytical solution of the appropriate similarity Eulerian equations is given in works [1–3].

At the same time it is known that consideration of one-dimensional unsteady gas flows of explosive type, especially numerical solution of the appropriate gas-dynamic equations is convenient to perform using Lagrangian mass coordinates. The Lagrangian description is naturally good for determining contact breaks. In this case, it is much easier to examine the kinetics of chemical reactions, processes of ionization and recombination in high-temperature products of explosion and the environment [4]. Radiation transfer may be also successfully analyzed in such an approach [5–6]. The present work contains solution of the problem of similarity strong explosion in the Lagrangian mass coordinates.

The gas dynamics equations in the Lagrangian mass variables have the form [7]:

$$\frac{1}{\rho} = \sigma_\nu r^{\nu-1}\frac{\partial r}{\partial m}; \qquad \frac{\partial u}{\partial t} + \sigma_\nu r^{\nu-1}\frac{\partial P}{\partial m} = 0; \qquad u = \frac{\partial r}{\partial t}; \tag{1}$$

$$\frac{\partial}{\partial t}\left(\varepsilon + \frac{u^2}{2}\right) + \sigma_\nu \frac{\partial}{\partial m}\left(r^{\nu-1}Pu\right) = 0; \qquad \frac{\partial}{\partial t}\left(\frac{P}{\rho^\gamma}\right) = 0. \tag{2}$$

where m– mass coordinate ($dm = \sigma_\nu \rho r^{\nu-1} dr$), ν – the symmetry factor, $\sigma_\nu = 2, 2\pi, 4\pi$ if $\nu = 1, 2, 3$, $\partial/\partial t$ – the substantive derivative. The coordinate of shock wave motion and its velocity are

$$r_F = \alpha^{-1/(\nu+2)}\left(E_0/\rho_0\right)^{1/(\nu+2)} t^{2/(\nu+2)}, \qquad m_F = \alpha^{-\nu/(\nu+2)}\frac{\sigma_\nu}{\nu}\left(\rho_0^2 E_0^\nu\right)^{1/(\nu+2)} t^{2\nu/(\nu+2)}, \qquad D = \frac{2}{\nu+2}\frac{r_F}{t}. \tag{3}$$

The conservation laws at front of strong SW give the boundary conditions:

$$\rho_F = \frac{\gamma+1}{\gamma-1}\rho_0 \qquad u_F = \frac{2}{\gamma+1}D \qquad P_F = \frac{2}{\gamma+1}\rho_0 D^2. \tag{4}$$

The functions determining gas-dynamic flow can be presented as:

$$r = r_F\eta(\xi), \quad u = \frac{2}{\gamma+1}D\,U(\xi), \quad \rho = \frac{\gamma+1}{\gamma-1}\rho_0 G(\xi), \quad P = \frac{2}{\gamma+1}\rho_o D^2\pi(\xi), \quad \xi = \frac{m}{m_F}. \tag{5}$$

The system of the differential equations for dimensionless functions has the following form:

$$\left.\begin{array}{l} \frac{\gamma-1}{\gamma+1}V = \nu\eta^{\nu-1}\frac{d\eta}{d\xi} \ \text{(a)} \qquad U + 2\xi\frac{dU}{d\xi} = 2\eta^{\nu-1}\frac{d\pi}{d\xi} \ \text{(b)} \qquad \frac{2}{\gamma+1}U = \eta - \nu\xi\frac{d\eta}{d\xi} \ \text{(c)} \\ \frac{d}{d\xi}\left[\xi\left(\pi V + U^2\right)\right] = 2\frac{d}{d\xi}\left(\eta^{\nu-1}\pi U\right) \ \text{(d)} \qquad \frac{d}{d\xi}\left(\xi\pi V^\gamma\right) = 0 \ \text{(e)} \end{array}\right\}. \tag{6}$$

In (6) $V(\xi) = 1/G(\xi)$ is the specific volume. Boundary conditions for similarity functions at SWF:

$$\xi = 1: \qquad \eta = 1 \qquad U = 1 \qquad G = 1 \qquad \pi = 1 \qquad V = 1 \tag{7}$$

The value of α from (3) depends on γ, ν and is determined by the law of energy conservation:

$$\alpha = \frac{\sigma_\nu}{\nu}\frac{2}{(\gamma+1)^2}\left(\frac{2}{\nu+2}\right)^2 \times \int_0^1 \left(\pi V + U^2\right) d\xi. \tag{8}$$

For plane explosion the solution is completely determined in parametric form in the following way

$$\left.\begin{array}{l} \xi = \left(3\frac{\gamma-1}{\gamma+1}x+1\right)^{F_1}\left(1-x^2\right)^{F_2}\left(\frac{1-x}{1+x}\right)^{F_3}, \qquad \Psi = 1-x^2, \qquad Z = \frac{\xi^{2(\gamma-2)/(\gamma+1)}}{\Psi^{(\gamma-1)/(\gamma+1)}} \\ U = \left(\frac{\xi^{\gamma-2}}{\Psi^{\gamma}}\right)^{\frac{1}{\gamma+1}}\left(1-\sqrt{1-\Psi}\right), \qquad V = (\xi Z)^{-1/(\gamma-1)}, \qquad \pi = Z/V \\ F_1 = -\frac{2}{3}\times\frac{9(\gamma^2-1)+(\gamma+1)^2}{9(\gamma-1)^2-(\gamma+1)^2}, \quad F_2 = \frac{6\gamma(\gamma-1)}{9(\gamma-1)^2-(\gamma+1)^2}, \quad F_3 = -\frac{2\gamma(\gamma+1)}{9(\gamma-1)^2-(\gamma+1)^2} \end{array}\right\}. \tag{9}$$

The expressions (9) hold true at any value of γ, with the exception of γ =2, at which the exponent F_i in (43) is the infinity. In this special case, we have

$$\left.\begin{array}{l} \xi = (1-x)^{2/3}\exp\left(-\frac{4}{3}\frac{x}{x+1}\right), \qquad \Psi = 1-x^2, \qquad Z = \Psi^{-1/3} \\ U = \Psi^{-2/3}\left(1-\sqrt{1-\Psi}\right), \qquad V = (\xi Z)^{-1}, \qquad \pi = Z/V = \xi\Psi^{-2/3} \end{array}\right\}. \tag{10}$$

The parameter x varies in an interval [0,1]. The value $x = 1$ corresponds to the symmetry plane, $x = 0$ obeys SWF. The total solution to the problem for the case of cylindrical symmetry:

$$\left.\begin{array}{ll} \xi = \left(\frac{2}{y+1}\right)\left(\frac{\gamma(y-1)+2}{2y}\right)^{\gamma/(2-\gamma)}, & U = \sqrt{\frac{y+1}{\gamma(y-1)+2}}\left(\frac{\gamma+1}{\gamma y+1}\right)^{(\gamma-1)/2\gamma}, \\ Z = \frac{y(y+1)}{\gamma(y-1)+2}\left(\frac{\gamma+1}{\gamma y+1}\right)^{(\gamma-1)/\gamma}, & v = \left(\frac{2y}{\gamma(y-1)+2}\right)^{2/(2-\gamma)}\left(\frac{\gamma y+1}{\gamma+1}\right)^{1/\gamma}, \\ \pi = \frac{\gamma+1}{2}\frac{y+1}{\gamma y+1}\left(\frac{\gamma(y-1)+2}{2y}\right)^{\gamma/(2-\gamma)}, & \eta = \frac{2}{\sqrt{y+1}\sqrt{\gamma(y-1)+2}}\left(\frac{\gamma y+1}{\gamma+1}\right)^{(\gamma+1)/2\gamma} \end{array}\right\}. \tag{11}$$

In the case of $\gamma = 2$, the following formulas are taken instead of (11)

$$\left.\begin{array}{l} \xi = \frac{2}{y+1}\exp[(1-y)/y], \qquad U = \sqrt{\frac{y+1}{2y}}\left(\frac{3}{2y+1}\right)^{1/4}, \qquad Z = \frac{y+1}{2}\sqrt{\frac{3}{2y+1}}, \\ v = \sqrt{\frac{2y+1}{3}}\exp[(y-1)/y], \quad \pi = \frac{3}{2}\frac{y+1}{2y+1}\exp[(1-y)/y], \quad \eta = \sqrt{\frac{2}{y(y+1)}}\left(\frac{2y+1}{3}\right)^{3/4} \end{array}\right\}. \tag{12}$$

Here the magnitude $y = 1$ corresponds to the front of SW. In vicinity of the symmetry axis value y tends to infinity.

For the spherical explosion it is possible to write the analytical solution only in the case of $\gamma = 2$.

References

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