SUPERSONIC FLOW OVER A WALL WITH TRANSVERSE AIR INJECTION

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Last time the certain interest has been given to the study of supersonic flows above surfaces in the case of perpendicular jet injection into the flow. One of the actual problems is to make the controlled efforts on moving surfaces at supersonic speed in the presence of gas jets injection to create the additional forces of braking.

In the present work the interaction problem of the supersonic free flow with a perpendicular jet injection through a nozzle on a wall is numerically simulated. The governing equations are the system of Navier-Stokes equations in the 2D case for a compressible turbulent gas in conservation form. In the Cartesian system of coordinates it has the following form:

$$\frac{\partial \mathbf{U}}{\partial t}+\frac{\partial \mathbf{E}(\mathbf{U})}{\partial x}+\frac{\partial \mathbf{F}(\mathbf{U})}{\partial z}=\frac{\partial \mathbf{V}_1(\mathbf{U},\mathbf{U}_x)}{\partial x}+\frac{\partial \mathbf{V}_2(\mathbf{U},\mathbf{U}_z)}{\partial x}+\frac{\partial \mathbf{W}_1(\mathbf{U},\mathbf{U}_x)}{\partial z}+\frac{\partial \mathbf{W}_2(\mathbf{U},\mathbf{U}_z)}{\partial z}, \qquad (1)$$

$$\mathbf{U}=\begin{bmatrix}\rho\\ \rho u\\ \rho w\\ E_t\end{bmatrix},\quad \mathbf{E}=\begin{bmatrix}\rho u\\ \rho u^2+P\\ \rho uw\\ (E_t+P)u\end{bmatrix},\quad \mathbf{F}=\begin{bmatrix}\rho w\\ \rho uw\\ \rho w^2+P\\ (E_t+P)w\end{bmatrix},\quad \mathbf{V}_2=\frac{1}{\mathrm{Re}}\left[0,-\frac{2}{3}\mu_t\frac{\partial w}{\partial z},\mu_t\frac{\partial u}{\partial z},\mu_t w\frac{\partial u}{\partial z}-\frac{2}{3}\mu_t u\frac{\partial w}{\partial z}\right]^T,$$

$$\mathbf{V}_1=\frac{1}{\mathrm{Re}}\left[0,\frac{4}{3}\mu_t\frac{\partial u}{\partial x},\mu_t\frac{\partial w}{\partial x},\mu_t w\frac{\partial w}{\partial x}+\frac{4}{3}\mu_t u\frac{\partial u}{\partial x}+\frac{\mu_t}{(\gamma-1)M_\infty^2\,\mathrm{Pr}}\frac{\partial T}{\partial x}\right]^T, \quad P=(\gamma-1)\left[E_t-\frac{1}{2}\left(\rho u^2+\rho w^2\right)\right],$$

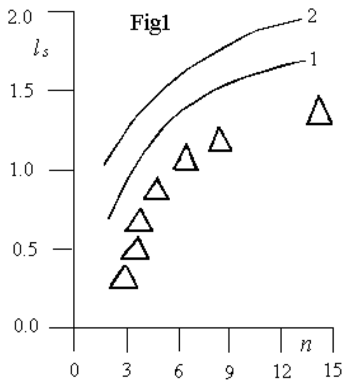
$$\mathbf{W}_1=\frac{1}{\mathrm{Re}}\left[0,\mu_t\frac{\partial w}{\partial x},-\frac{2}{3}\mu_t\frac{\partial u}{\partial x},\mu_t u\frac{\partial w}{\partial x}-\frac{2}{3}\mu_t w\frac{\partial u}{\partial x}\right]^T, \quad T=\left(\frac{1}{\rho c_v}\right)\left[E_t-\frac{1}{2}\left(\rho u^2+\rho w^2\right)\right],$$

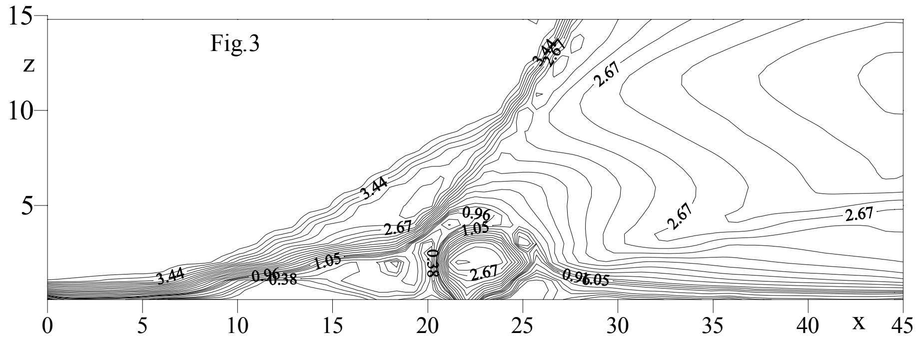
$$\mathbf{W}_2=\frac{1}{\mathrm{Re}}\left[0,\mu_t\frac{\partial u}{\partial z},\ \frac{4}{3}\mu_t\frac{\partial w}{\partial z},\mu_t u\frac{\partial u}{\partial z}+\frac{4}{3}\mu_t w\frac{\partial w}{\partial z}+\frac{\mu_t}{(\gamma-1)M_\infty^2\,\mathrm{Pr}}\frac{\partial T}{\partial z}\right]^T, \quad c_v=\frac{1}{\gamma(\gamma-1)M_\infty^2}.$$

Here the following symbols are applied: u, w are the velocities components in the x and z directions; ρ is the density; P is the pressure; T is the temperature; c_v is the specific heat capacity of the mixture with the constant volume; γ is the adiabat index; M_0 and M_∞ are the Mach number of a jet and flow; μ_t is the coefficient of the vortex turbulent viscosity; Re, Pr are the Reynolds and Prandtl number. The index ∞ concerns to the values of flow parameters, the index 0 - to values of jet parameters.

The turbulent dynamic viscosity coefficient was determined by Baldwin – Lomax model [1].
Equations (1) were integrated under the following boundary conditions: on an inflow the parameters of a flow $u=1$, $w=0$, $\rho=1$, $T=1$; on a wall a condition of adiabatic no-slip wall ; on a nozzle the parameters of a jet $u=0$, $T=0.6$ $w=\sqrt{T}M_0/M_\infty$, $P_0=nP_\infty$, here the $n=P_0/P_\infty$ - degree of pressure ratio , where P_0 - pressure in a jet, P_∞ - pressure of a flow ; in inflow section near a wall is set the boundary layer, the streamwise velocity approximated by the degree law [2]; on the top and outflow border the non-reflecting conditions [3].

To facilitate griding of complex geometries (in a boundary layer and at a level of a nozzle) a coordinate transformation to a generalized $\xi=\xi(x)$ $\eta=\eta(z)$ coordinate system is performed. To solve of system (1) the Beam-Warming scheme is used.





In figures 1-3 some numerical results of simulation at parameters $M_\infty = 4$, $n = 4 \div 10$ and various values of Mach number of a jet are shown.

In front of a jet due to braking of a flow the pressure raises and therefore a head shock wave is formed. The interaction of this shock wave with a boundary layer and the presence of a counter gradient of pressure give the vortex zone in front of a jet. The dependence of the lengths of a vortex zone on the degree of the pressure ratio is shown in figure 1 for a sonic jet with $M_\infty = 3$, $\Pr = 0.9$, $\mathrm{Re} = 10^7$ (curve 1- length of a vortex zone l_s for $M_0 = 1$, curve 2- for $M_0 = 1.5$, « $\Delta\,\Delta$ » - experimental researches in a spatial case in work [4]). The results show that in a qualitative sense numerical results of the dependence of l_s on the degree of the pressure ratio corresponds to experiment. In the quantitative respect, the length of a vortex zone in a flat case it is more than in spatial, because there is explained by absence lateral spread.

The force of interaction is determined from wall pressure distribution by the integral $F = \int_{l_s} (P - P_\infty) dl_s$.

Figure 2 shows the curve of change forces of interaction depending on Mach number of a jet (curve 1- $n = 4$, curve 2- $n = 10$). So at increasing a jet Mach number from $M_0 = 1$ to $M_0 = 2$ for a case with degree of pressure ratio $n = 10$ the force of interaction is increased at 1.58, while for $n = 4$ forces of interaction change only at 1.32.

The relative size of local Mach number $M = \sqrt{u^2 + w^2}/c$, where c is local velocities of a sound for $M_0 = 2$, $n = 10$ is shown on a figure 3. From the diagram follows, that a supersonic injection is accompanied by occurrence of a so-called barrel. For border of the formed barrel $M = 1$ corresponds. The value of Mach number increases up to $M = 3$ inside it. Behind a barrel the flow go down, the Mach number decreases up to 0,9. Then the flow again arrives in a supersonic zone.

References

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