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Similarity solution for one-dimensional strong explosion in the perfect gas

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Abstract

The present work is the research of similarity solution for a one-dimensional strong explosion in the perfect gas. The analysis of the equations describing gas-dynamic flow is made in the Lagrangian mass coordinates. The calculation results for flows with different symmetry are described. An exact analytical solution to the problem on plane and cylindrical geometry is given.

1. Introduction

The strong explosion theory has arisen from the necessity to describe in the environment the distributions of shock waves caused by explosion of charges, having large specific energy both in small weight and in volume. This theory was developed in the works of L.I. Sedov, and also G. Taylor, K.P. Stanjukovich and J. Neumann. The analytical solution of the appropriate similarity Eulerian equations is given in works [1–3].

At the same time it is known that consideration of one-dimensional unsteady gas flows of explosive type, especially numerical solution of the appropriate gas-dynamic equations is convenient to perform using Lagrangian mass coordinates. The Lagrangian description is naturally good for determining contact breaks. In this case, it is much easier to examine the kinetics of chemical reactions, processes of ionization and recombination in high-temperature products of explosion and the environment [4]. Radiation transfer may be also successfully analyzed in such an approach [5–6].

The present work contains the analysis of the problem of strong explosion in perfect gases. Its solution is considered in the Lagrangian mass coordinates.

2. Problem statement

Let us consider a one-dimensional strong explosion in the environment with a constant density ρ_0. Gas flow, as is known, is described by conservation laws of mass, momentum, and energy which in the Eulerian variables have the following form:

equation of continuity:

$$\frac{\partial\rho}{\partial t}+\frac{\partial(\rho u)}{\partial r}+(\nu-1)\frac{\rho u}{r}=0 \tag{1}$$

motion equation:

$$\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial r}+\frac{1}{\rho}\frac{\partial P}{\partial r}=0 \tag{2}$$

energy equation:

$$\frac{\partial}{\partial t}\left[\rho\left(\varepsilon+\frac{u^2}{2}\right)\right]+\frac{1}{r^{\nu-1}}\frac{\partial}{\partial r}\left[r^{\nu-1}\rho u\left(\varepsilon+\frac{P}{\rho}+\frac{u^2}{2}\right)\right]=0 \tag{3}$$

Besides, if the motion in the field of explosion is adiabatic, then the equation is valid

$$\frac{\partial}{\partial t}\left(\frac{P}{\rho^\gamma}\right)+u\frac{\partial}{\partial r}\left(\frac{P}{\rho^\gamma}\right)=0\,, \tag{4}$$

This equation specifies the constancy of entropy for gas particles. In (1) r is the space variable, t the time, ν the symmetry factor, ρ the density, P the pressure, u the mass velocity, ε the internal energy that is determined by the state equation for a perfect gas with constant adiabatic coefficient γ:

$$\varepsilon = \frac{P/\rho}{\gamma - 1} \tag{5}$$

Let energy of explosion be equal to E_0 on the unit of the area in the plane explosion case, on the unit of the length for cylindrical symmetry and full energy for spherical geometry. In the case of strong explosion the gas pressure in the unperturbed area is negligible in comparison with the pressure behind the front of a shock wave. Therefore in the considered problem there are no other dimensional parameters, except for E_0 and ρ_0 (their dimensions are $[E_0] = g\ cm^{\nu-1}/\sec^2$, $[\rho_0] = g/cm^3$). The unique combination having the lengths dimensionality which can be made of the values E_0, ρ_0 t is

$$r_F = (E/\rho_0)^{\frac{1}{\nu+2}} t^{\frac{2}{\nu+2}} \tag{6}$$

Here the magnitude E is proportional to E_0, and r_F is the coordinate of the shock wave front (SWF).

$$E_0 = \alpha E \tag{7}$$

Thus, the unique dimensionless coordinate is

$$\lambda = \frac{r}{r_F} \tag{8}$$

In such a statement the problem is the similarity case. The external border of the perturbed area is the front of a shock wave where the conservation laws hold true:

$$\rho_F = \frac{\gamma+1}{\gamma-1}\rho_0 \qquad u_F = \frac{2}{\gamma+1}D \qquad P_F = \frac{2}{\gamma+1}\rho_0 D^2 \tag{9}$$

In (9) the subscript F marks the gas parameters at FSW. The velocity of the shock wave extension D is defined through the FSW coordinate:

$$D = \frac{dr_F}{dt} = \frac{2}{\nu+2}\frac{r_F}{t} \qquad 10)$$

Below we shall consider the solution of a given problem in the Lagrangian variables.

3. Analysis of the Lagrangian equations

The mass coordinate is determined by the expression

$$dm = \sigma_\nu \rho r^{\nu-1} dr \tag{11}$$

Here the factor σ_ν depends on the symmetry:

$$\sigma_\nu = \begin{cases} 2 & \nu = 1 \\ 2\pi & \nu = 2 \\ 4\pi & \nu = 3 \end{cases} \tag{12}$$

If the value m is calculated from the plane (the axis or the center) symmetry the mass coordinate of a point with a given Eulerian coordinate r is the mass of the gas in the area $[0, r]$. The gas dynamics equations in the Lagrangian mass variables have the form [7]:

$$\frac{1}{\rho} = \sigma_\nu r^{\nu-1}\frac{\partial r}{\partial m}; \qquad \frac{\partial u}{\partial t} + \sigma_\nu r^{\nu-1}\frac{\partial P}{\partial m} = 0; \qquad u = \frac{\partial r}{\partial t}; \tag{13}$$

$$\frac{\partial}{\partial t}\left(\varepsilon + \frac{u^2}{2}\right) + \sigma_\nu \frac{\partial}{\partial m}\left(r^{\nu-1}Pu\right) = 0; \qquad \frac{\partial}{\partial t}\left(\frac{P}{\rho^\gamma}\right) = 0. \tag{14}$$

The symbol $\partial/\partial t$ in (14) - (15) implies the substantive derivative $\partial/\partial t + u\partial/\partial r$. As SWF moves on the gas with constant density the mass coordinate of the front according to (11) and (6) is

$$m_F = \frac{\sigma_\nu}{\nu}\rho_0 r_F^\nu = \frac{\sigma_\nu}{\nu}\left(\rho_0^2 E^\nu\right)^{\frac{1}{\nu+2}} t^{\frac{2\nu}{\nu+2}} \tag{15}$$

The similarity dimensionless mass coordinate is determined as follows

$$\xi = \frac{m}{m_F} \tag{16}$$

The functions determining gas-dynamic flow can be presented as:

$$r = r_F \eta(\xi) \qquad u = \frac{2}{\gamma+1} D\, U(\xi) \qquad \rho = \frac{\gamma+1}{\gamma-1}\rho_0 G(\xi) \qquad P = \frac{2}{\gamma+1}\rho_o D^2 \pi(\xi) \tag{17}$$

The specific volume is the inverse value to ρ:

$$v = \frac{1}{\rho} = \frac{\gamma-1}{\gamma+1}\frac{1}{\rho_0} V(\xi) \tag{18}$$

Substituting (17) - (18) into (13) and (14) yields the system of the differential equations for dimensionless functions - representatives:

$$\left.\begin{array}{l} \frac{\gamma-1}{\gamma+1} V = \nu \eta^{\nu-1}\frac{d\eta}{d\xi} \quad (a) \qquad U + 2\xi \frac{dU}{d\xi} = 2\eta^{\nu-1}\frac{d\pi}{d\xi} \quad (b) \qquad \frac{2}{\gamma+1} U = \eta - \nu\xi\frac{d\eta}{d\xi} \quad (c) \\ \frac{d}{d\xi}\left[\xi\left(\pi V + U^2\right)\right] = 2\frac{d}{d\xi}\left(\eta^{\nu-1}\pi U\right) \quad (d) \qquad \frac{d}{d\xi}\left(\xi\pi V^{\gamma}\right) = 0 \quad (e) \end{array}\right\} \tag{19}$$

Expressions (19) are accordingly the equations of continuity, motion, the equation for mass velocity, the conservation law of energy and the requirement of adiabatic flow.
From equations (9) it is possible to find boundary conditions for similarity functions at SW front

$$\xi = 1: \qquad \eta = 1 \qquad U = 1 \qquad G = 1 \qquad \pi = 1 \qquad V = 1 \tag{20}$$

Integrating equations (19d) and (19e) in view of boundary conditions (20), we obtain two algebraic equations connecting the flow characteristics. The first of them is the law of conservation of energy

$$\xi\left(\pi V + U^2\right) = 2\eta^{\nu-1}\pi U \tag{21}$$

and the second – the constancy of the entropy of a mass particle behind SWF

$$\xi\pi V^{\gamma} = 1 \tag{22}$$

So, the problem reduces to integration of the system of the equations:

$$\left.\begin{array}{l} \frac{\gamma-1}{\gamma+1} V = \nu \eta^{\nu-1}\frac{d\eta}{d\xi}, (a) \qquad U + 2\xi \frac{dU}{d\xi} = 2\eta^{\nu-1}\frac{d\pi}{d\xi}, (b) \qquad \frac{2}{\gamma+1} U = \eta - \nu\xi\frac{d\eta}{d\xi}, (c) \\ \xi\left(\pi V + U^2\right) = 2\eta^{\nu-1}\pi U, (d) \qquad \xi\pi V^{\gamma} = 1\, (e) \end{array}\right\} \tag{23}$$

with boundary conditions (20).

As in the considered approximation the internal energy of the gas before the wave front according to (5) is equal to zero ($\varepsilon_0 = 0$), the energy of the area entrained in motion is constant and equal to the explosion energy E_0. From here we obtain:

$$E_0 = \int_0^{m_F}\left(\varepsilon + \frac{u^2}{2}\right) dm = m_F \frac{2D^2}{(\gamma+1)^2}\int_0^1 (\pi V + U^2) d\xi \tag{24}$$

The last relation with account of (7), (10) defines the constant value of α from (7) which depends on γ, ν and is determined by the gas-dynamic profiles of flow in the area of explosion:

$$\alpha = \frac{\sigma_\nu}{\nu}\frac{2}{(\gamma+1)^2}\left(\frac{2}{\nu+2}\right)^2 \int_0^1 \left(\pi V + U^2\right) d\xi, \tag{25}$$

For the analysis of system (23) in the general case it is necessary to introduce a new function

$$Z = \pi V \tag{26}$$

and with the help of two last equations (23) to exclude from the equations the variables V and η

$$V = (\xi Z)^{-1/(\gamma-1)}, \qquad \eta^{\nu-1} = \frac{Z + U^2}{2Z^2 U}(\xi Z)^{-(2-\gamma)/(\gamma-1)} \tag{27}$$

In total, system (23) can reduce to two differential equations

$$\frac{dU}{d\ln\xi} = \frac{c_1 b_2 - c_2 b_1}{a_1 b_2 - a_2 b_1}, \qquad \frac{dZ}{d\ln\xi} = \frac{a_1 c_2 - a_2 c_1}{a_1 b_2 - a_2 b_1} \tag{28}$$

where the coefficients a, b, c are:

$$a_1 = \nu\left[(Z+U^2)^2 + (\gamma-1)Z(Z-U^2)\right], \quad b_1 = \nu(\gamma-1)U^3, \quad c_1 = (\gamma-1)ZU(Z+U^2)$$
$$a_2 = 2(\gamma-1)ZU, \quad b_2 = -\gamma(Z+U^2), \quad c_2 = Z^2 + (2-\gamma)ZU^2 \tag{29}$$

A further analysis is standard. Dividing one equation from (28) by another yields

$$\frac{dZ}{dU} = \frac{\nu\left[(Z+U^2)^2 + (\gamma-1)Z(Z-U^2)\right]\left[Z^2 + (2-\gamma)ZU^2\right] - 2(\gamma-1)^2 Z^2 U^2 (Z+U^2)}{-\gamma(\gamma-1)ZU(Z+U^2)^2 - \nu(\gamma-1)U^3\left[Z^2 + (2-\gamma)ZU^2\right]} \tag{30}$$

Equation (30) can be simplified if a new variable y is introduced

$$y = \frac{Z}{U^2} \tag{31}$$

Then it will become

$$U\frac{dy}{dU} = \frac{d_0}{\gamma-1} \times \frac{y^3 + ry^2 + sy + t}{\gamma(y+1)^2 + \nu(y+2-\gamma)} \tag{32}$$

where

$$r = \frac{d_1}{d_0}, \qquad s = \frac{d_2}{d_0}, \qquad t = \frac{d_3}{d_0}, \tag{33}$$

coefficients d_0, d_1, d_2, d_3 are accordingly equal to

$$\left.\begin{aligned} d_0 &= -[2\gamma(\gamma-1) + \nu\gamma] \\ d_1 &= -[4\gamma(\gamma-1) + 2\nu(\gamma-1) + \nu\{\gamma(2-\gamma) + 3 - \gamma\} - 2(\gamma-1)^2] \\ d_2 &= -[2\gamma(\gamma-1) + 2\nu(\gamma-1)(2-\gamma) + \nu\{1 + (3-\gamma)(2-\gamma)\} - 2(\gamma-1)^2] \\ d_3 &= -\nu(2-\gamma) \end{aligned}\right\} \tag{34}$$

If the change in the variable $y = x - r/3$ is made, instead of (32) a more simple equation is obtained:

$$U\frac{dx}{dU} = \frac{d_0}{\gamma-1} \times \frac{x^3 + Px + Q}{\gamma(x+1-r/3)^2 + \nu(x - r/3 + 2 - \gamma)} \tag{35}$$

Here the following designations are accepted

$$P = s - \frac{r^2}{3}, \qquad Q = \frac{sr}{3} - t - \frac{2}{27}r^3 \tag{36}$$

Solving the last equation allows obtaining the dependence $U(x)$, i.e. $Z(U)$. Substituting this expression into (28) and integrating these equations determines all functions from the variable ξ.

Plane explosion

It is seen that equation (35) is complicated in character. However it should not be always solved. In the case of a plane explosion ($\nu = 1$) the system of equations (23) becomes essentially simpler. The Eulerian coordinate r is absent in the equations, and equation (23c) allows calculating the value r. Then instead of (23) we have the following

$$\left.\begin{aligned} &2\frac{dU}{d\xi} = (\xi Z)^{-1/(\gamma-1)}\left(1 + \frac{\xi}{Z}\frac{dZ}{d\xi}\right), \text{(a)} \qquad U + 2\xi\frac{dU}{d\xi} = \frac{2}{\gamma-1}\xi^{[1/(\gamma-1)-1]}Z^{\gamma/(\gamma-1)}\left(1 + \frac{\gamma\xi}{Z}\frac{dZ}{d\xi}\right), \text{(b)} \\ &Z + U^2 = 2Z^2U(\xi Z)^{[1/(\gamma-1)-1]} \quad \text{(c)} \end{aligned}\right\} \tag{37}$$

Solving quadratic equation (37c), we find the velocity dependence on ξ and Z:

$$U = \frac{Z}{\xi}(\xi Z)^{\frac{1}{\gamma-1}}\left[1 - \sqrt{1 - \frac{\xi^2}{Z}(\xi Z)^{-\frac{2}{\gamma-1}}}\right] \tag{38}$$

The sign minus before the root is chosen to obey the condition $U(0) = 0$. Further from system (37) - (38) it is easy to obtain the equation determining the dependence of one of the functions on the similarity coordinate:

$$\frac{dZ}{d\xi}=\frac{Z}{\xi}\times\frac{\left[\frac{2}{\gamma-1}-\frac{\xi^2}{Z}(\xi Z)^{-2/(\gamma-1)}\right]-\left[1-\sqrt{1-\frac{\xi^2}{Z}(\xi Z)^{-2/(\gamma-1)}}\right]}{\frac{\xi^2}{Z}(\xi Z)^{-2/(\gamma-1)}-\frac{2\gamma}{\gamma-1}} \quad (39)$$

To simplify the last expression, let us introduce a new variable Ψ

$$\Psi=\frac{\xi^2}{Z}(\xi Z)^{-2/(\gamma-1)}=\frac{\xi^{2(\gamma-2)/(\gamma-1)}}{Z^{(\gamma+1)/(\gamma-1)}} \quad (40)$$

The equation as it follows from (39) has the form

$$\frac{d\Psi}{d\xi}=\frac{\Psi}{\xi}\times\frac{2\frac{2\gamma-1}{\gamma-1}-3\Psi-\frac{\gamma+1}{\gamma-1}\left[1-\sqrt{1-\Psi}\right]}{\frac{2\gamma}{\gamma-1}-\Psi} \quad (41)$$

If the variables $1-\Psi=x^2$ are substituted, instead of (41) we have

$$\frac{1}{2\xi}\frac{d\xi}{dx}=\frac{\frac{\gamma+1}{\gamma-1}+x^2}{(x^2-1)\left(\frac{\gamma+1}{\gamma-1}+3x\right)} \quad (42)$$

The solution of equation (42) subject to boundary conditions (20) takes the form

$$\left.\begin{array}{l}\xi=\left(3\frac{\gamma-1}{\gamma+1}x+1\right)^{F_1}\left(1-x^2\right)^{F_2}\left(\frac{1-x}{1+x}\right)^{F_3}, \qquad \Psi=1-x^2, \qquad Z=\frac{\xi^{2(\gamma-2)/(\gamma+1)}}{\Psi^{(\gamma-1)/(\gamma+1)}} \\ U=\left(\frac{\xi^{\gamma-2}}{\Psi^{\gamma}}\right)^{\frac{1}{\gamma+1}}\left(1-\sqrt{1-\Psi}\right), \qquad V=\left(\xi Z\right)^{-1/(\gamma-1)}, \qquad \pi=Z/V \\ F_1=-\frac{2}{3}\times\frac{9(\gamma^2-1)+(\gamma+1)^2}{9(\gamma-1)^2-(\gamma+1)^2}, \quad F_2=\frac{6\gamma(\gamma-1)}{9(\gamma-1)^2-(\gamma+1)^2}, \quad F_3=-\frac{2\gamma(\gamma+1)}{9(\gamma-1)^2-(\gamma+1)^2}\end{array}\right\} \quad (43)$$

Obviously, expressions (43) hold true at any value of γ, with the exception of γ =2, at which the exponent F_i in (43) is the infinity. In this special case, we have

$$\left.\begin{array}{l}\xi=(1-x)^{2/3}\exp\left(-\frac{4}{3}\frac{x}{x+1}\right), \qquad \Psi=1-x^2, \qquad Z=\Psi^{-1/3} \\ U=\Psi^{-2/3}\left(1-\sqrt{1-\Psi}\right), \qquad V=\left(\xi Z\right)^{-1}, \qquad \pi=Z/V=\xi\Psi^{-2/3}\end{array}\right\} \quad (44)$$

Formulas (43)–(44) in parametric form completely determine the solution. Note that it continuously depends on the adiabatic coefficient γ in the vicinity $\gamma=2$. The parameter x varies in an interval [0,1]. The value $x=1$ corresponds to the symmetry plane, $x=0$ obeys SWF. The Eulerian coordinate is determined by integrating equation (19a)

$$\eta=\frac{\gamma-1}{\gamma+1}\int_0^{\xi}V(\xi)d\xi \quad (45)$$

It is not difficult to indicate that near the symmetry plane the solution depends on the similarity variable as follows:

$$U\sim\xi^{(\gamma-1)/\gamma}, \qquad Z\sim\xi^{-1/\gamma}, \qquad V\sim\xi^{-1/\gamma}, \qquad \pi\sim\xi^0, \qquad \eta\sim\xi^{(\gamma-1)/\gamma} \quad (46)$$

Axial-symmetric explosion

In this case, the values d_0, d_1, d_2, d_3 from (34) have the form:

$$d_0=-2\gamma^2, \qquad d_1=-6\gamma, \qquad d_2=2(\gamma^2-2\gamma-2), \qquad d_3=2(\gamma-2) \quad (47)$$

Accordingly, the coefficients r, s, t included into equation (32) are equal to

$$r = \frac{3}{\gamma}, \qquad s = -\frac{\gamma^2 - 2\gamma - 2}{\gamma^2}, \qquad t = -\frac{\gamma - 2}{\gamma^2} \tag{48}$$

and the factors P and Q from (35) are:

$$P = -\left(\frac{\gamma - 1}{\gamma}\right)^2, \qquad Q = 0 \tag{49}$$

As a result, the solution of equation (35) satisfying boundary condition (20) ($U = 1$ if $x = (\gamma + 1)/\gamma$) can be presented as

$$U = \frac{1}{\sqrt{\gamma}}\left(\frac{\gamma + 1}{\gamma}\right)^{(\gamma-1)/2\gamma} \sqrt{\frac{x + (\gamma - 1)/\gamma}{x - (\gamma - 1)/\gamma}}\; x^{-(\gamma-1)/2\gamma} \tag{50}$$

After this, it is easy to find a total solution to the problem for the case of cylindrical symmetry:

$$\left.\begin{aligned}
&\xi = \left(\frac{2}{y+1}\right)\left(\frac{\gamma(y-1)+2}{2y}\right)^{\gamma/(2-\gamma)}, && U = \sqrt{\frac{y+1}{\gamma(y-1)+2}\left(\frac{\gamma+1}{\gamma y+1}\right)^{(\gamma-1)/2\gamma}},\\
&Z = \frac{y(y+1)}{\gamma(y-1)+2}\left(\frac{\gamma+1}{\gamma y+1}\right)^{(\gamma-1)/\gamma}, && v = \left(\frac{2y}{\gamma(y-1)+2}\right)^{2/(2-\gamma)}\left(\frac{\gamma y+1}{\gamma+1}\right)^{1/\gamma},\\
&\pi = \frac{\gamma+1}{2}\frac{y+1}{\gamma y+1}\left(\frac{\gamma(y-1)+2}{2y}\right)^{\gamma/(2-\gamma)}, && \eta = \frac{2}{\sqrt{y+1}\sqrt{\gamma(y-1)+2}}\left(\frac{\gamma y+1}{\gamma+1}\right)^{(\gamma+1)/2\gamma}
\end{aligned}\right\} \tag{51}$$

In the case of $\gamma = 2$, the following formulas are taken instead of (51)

$$\left.\begin{aligned}
&\xi = \frac{2}{y+1}\exp[(1-y)/y], \quad U = \sqrt{\frac{y+1}{2y}\left(\frac{3}{2y+1}\right)^{1/4}}, \quad Z = \frac{y+1}{2}\sqrt{\frac{3}{2y+1}},\\
&v = \sqrt{\frac{2y+1}{3}}\exp[(y-1)/y], \quad \pi = \frac{3}{2}\frac{y+1}{2y+1}\exp[(1-y)/y], \quad \eta = \sqrt{\frac{2}{y(y+1)}\left(\frac{2y+1}{3}\right)^{3/4}}
\end{aligned}\right\} \tag{52}$$

The asymptotic behavior of the gas dynamic profiles near the symmetry axis is as follows

$$U \sim \xi^{(\gamma-1)/\gamma}, \qquad Z \sim \xi^{-1/\gamma}, \qquad V \sim \xi^{-1/\gamma}, \qquad \pi \sim \xi^0, \qquad \eta \sim \xi^{(\gamma-1)/\gamma} \tag{53}$$

The point explosion

For a spherical explosion equation (35) can be also presented as

$$U\frac{dx}{dU} = \frac{d_0}{\gamma - 1} \times \frac{(x - x_1)(x - x_2)(x - x_3)}{\gamma(x + 1 - r/3)^2 + \nu(x - r/3 + 2 - \gamma)} \tag{54}$$

However, it is not possible to determine the analytical dependence of the roots x_1, x_2, x_3 on the value of γ. Therefore, it's solving and a further analysis should be made numerically. Only in the case of $\gamma = 2$ it is possible to write the analytical solution:

$$\left.\begin{aligned}
&\xi = \left(\frac{2}{y+1}\right)^{6/5}\exp\left[\frac{6(1-y)}{5y}\right], && U = \left(\frac{y+1}{2}\right)^{3/5}\left(\frac{3}{y(2y+1)}\right)^{2/5},\\
&Z = y\left(\frac{y+1}{2}\right)^{6/5}\left(\frac{3}{y(2y+1)}\right)^{4/5}, && v = \frac{1}{y}\left(\frac{y(2y+1)}{3}\right)^{4/5}\exp\left[\frac{6(1-y)}{5y}\right],\\
&\pi = y^2\left(\frac{y+1}{2}\right)^{6/5}\left(\frac{3}{y(2y+1)}\right)^{8/5}\exp\left[\frac{6(1-y)}{5y}\right], && \eta = \frac{1}{y}\left(\frac{2}{y+1}\right)^{2/5}\left(\frac{y(2y+1)}{3}\right)^{3/5}
\end{aligned}\right\} \tag{55}$$

4. Results and discussion

Some calculation results of a strong one-dimensional explosion in the perfect gas are given below. Fig. 1 shows the gas-dynamic profiles depending on the Lagrangian mass coordinate for the plane blast.

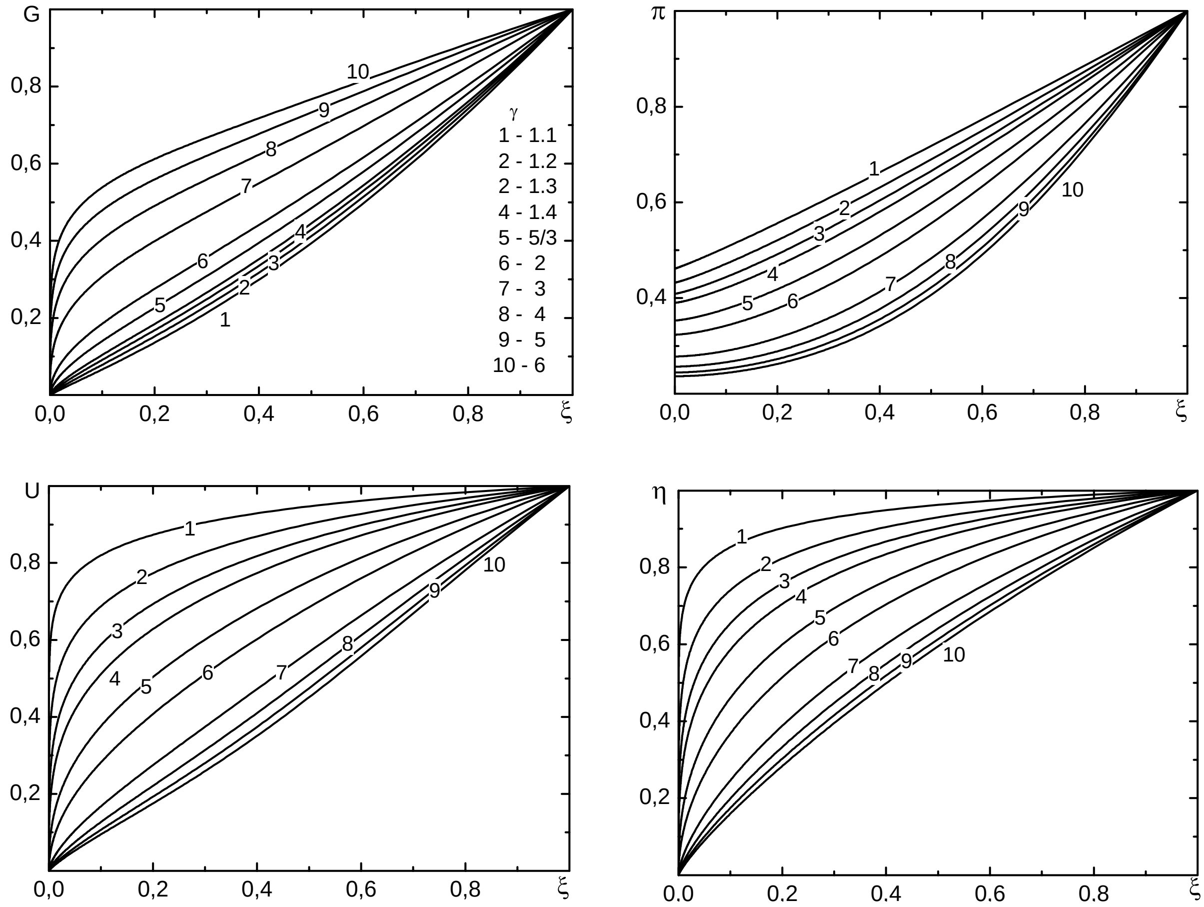

Fig.1 Distributions of density G, pressure π, mass velocity U and Eulerian coordinate η in the line of ξ.

Fig. 2 presents the dependence of the dimensionless parameter α, that determines according to (6)-(7) the SWF motion law, on the γ $r_F = \alpha^{-1/3}(E_0/\rho_0)^{1/3} t^{2/3} = \beta(E_0/\rho_0)^{1/3} t^{2/3}$. The proportion between the thermal f_T and kinetic energy f_K of a moving gas is given in fig. 3

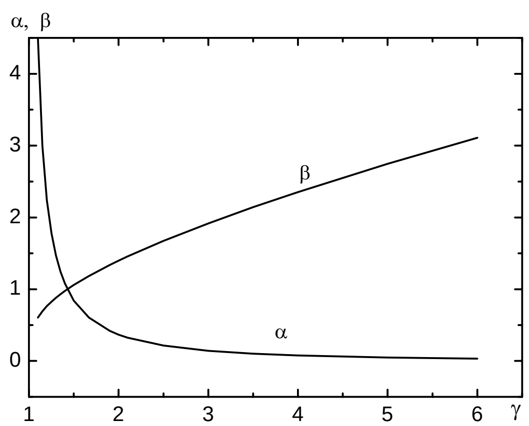

Fig. 2 The dependence of α and β on γ for $\nu = 1$

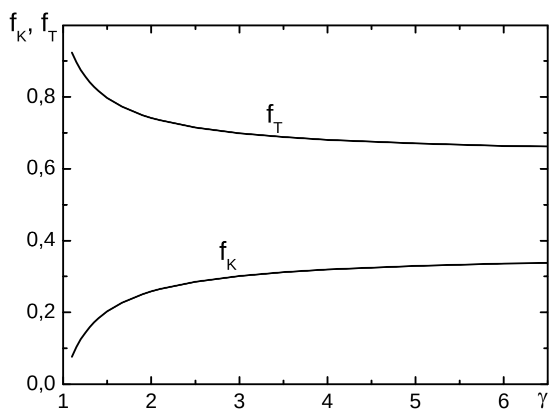

Fig. 3 The thermal and kinetic energy for plane blast

The distributions of gas density, pressure and velocity along the mass coordinate for cylindrical and spherical explosion are shown in figs. 4, 5.

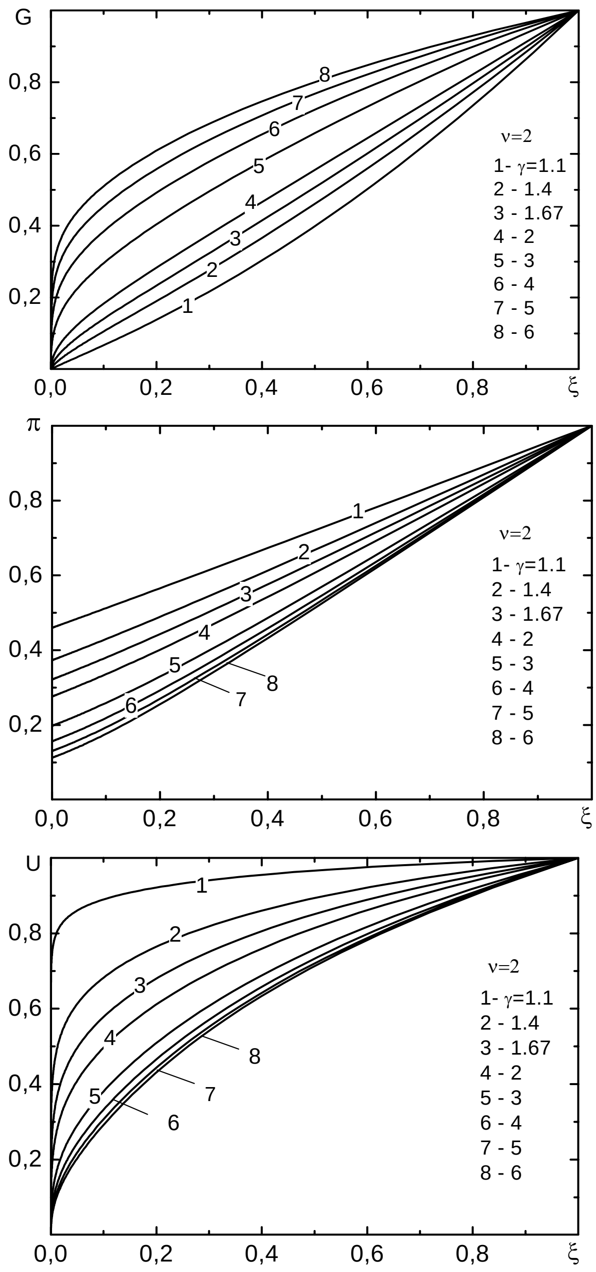

Fig. 4 Profiles of gas-dynamic parameters for $\nu = 2$

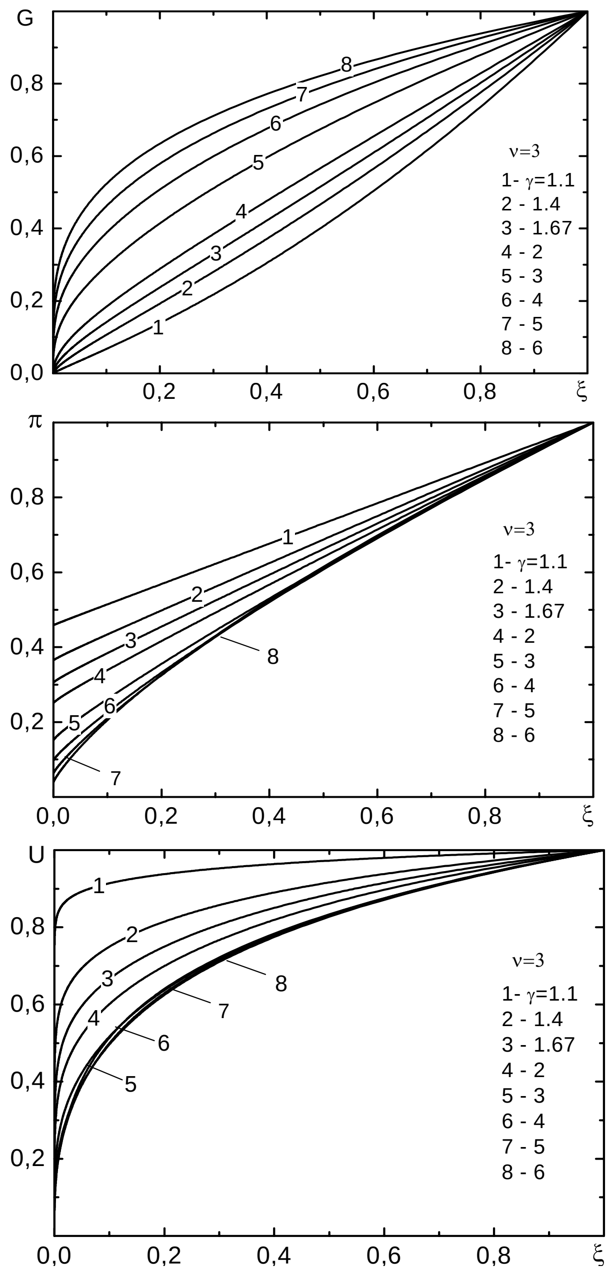

Fig. 5 Profiles of gas-dynamic parameters for $\nu = 3$

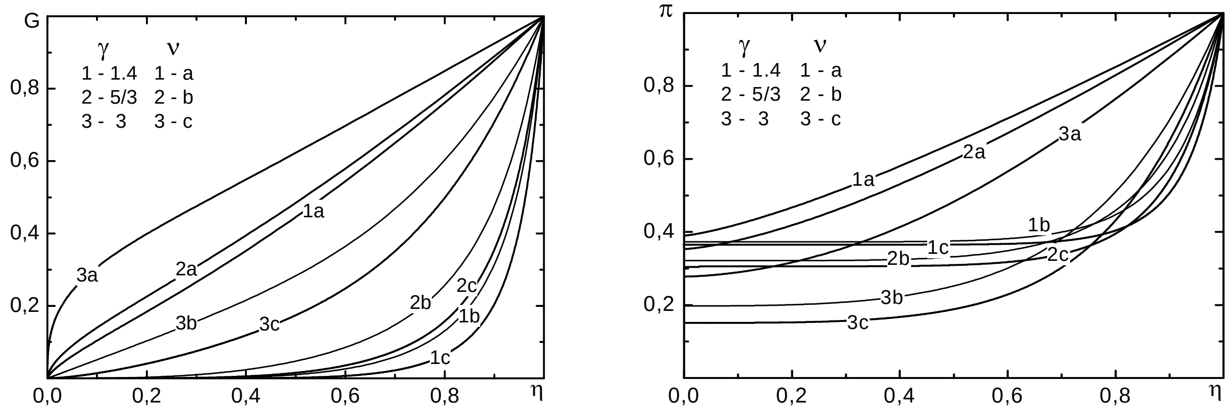

Fig. 6 Distributions of density and pressure as a function of the Eulerian similarity coordinate

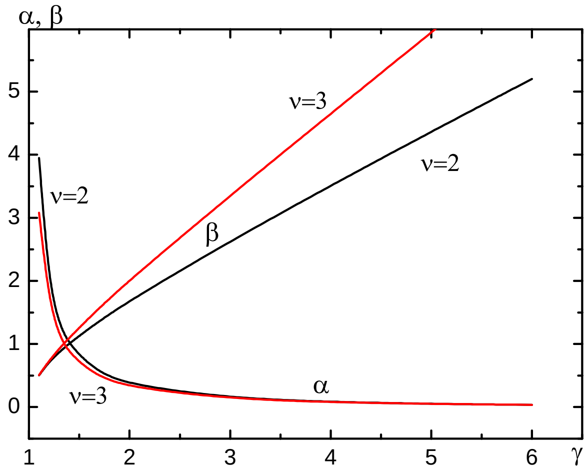

Fig.7 Magnitudes of α and β as functions of γ, ν

The dimensionless functions describing the flow of perfect the gas at one-dimensional strong explosion in the Eulerian coordinate are plotted in fig. 6.
At last, fig. 7 shows the dependence of the values of α and $\beta = \alpha^{-\nu/(\nu+2)}$ on the adiabatic coefficient γ and the symmetry factor ν.

5. Conclusion

Analytical solutions of the gas-dynamics equations in the Lagrangian mass coordinates that describe the flow of a perfect gas at a one-dimensional strong explosion are obtained. The analysis of the gas flow for different adiabatic coefficient γ and symmetry parameter ν is made.

Acknowledgements

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