INTERACTION OF SMALL HYDRODYNAMIC PERTURBATIONS WITH A NONEQUILIBRIUM REGION IN A GAS FLOW

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The problem of interaction of small hydrodynamic perturbations with a nonequilibrium region in a gas flow with different models of energy pumping is solved. One-dimensional and two-dimensional interactions are considered. A range of system parameters is found in which interaction occurs in a resonant manner (significant amplification of perturbations is observed). It is demonstrated that interaction of vortex perturbations with the nonequilibrium region generates heat waves.

Key words: *nonequilibrium gas, gas-flow CO_2 laser.*

Introduction. An important issue for continuous gas-flow lasers is hydrodynamic stability of the active medium. The starting point in the stability analysis is consideration of interaction of small hydrodynamic perturbations with a nonequilibrium region in the flow. Amplification of perturbations can be considered as the onset of instability.

Gas-flow systems with a limited volume of the nonequilibrium gas region have two more factors, which are usually ignored in studying the evolution of small perturbations. The first factor is a localized energy contribution, which ensures the existence of a nonequilibrium region responsible for origination of gradients of thermodynamic parameters in the flow. The second factor is downstream entrainment of growing perturbations to the heated equilibrium region. The latter does not refer to acoustic perturbations. Acoustic waves can be reflected in a medium with gradients; in this case, the inhomogeneous zone itself can act as a resonator where the perturbations are amplified.

The objective of the present work is to study the evolution of small hydrodynamic perturbations in a nonequilibrium region through which an initially equilibrium gas passes and to determine the range of perturbation frequencies in which the nonequilibrium region starts operating as a resonator.

Formulation of the Problem. Regions of nonequilibrium states in terms of vibrational degrees of freedom are generated in a one-dimensional nonequilibrium gas flow by means of energy input. Two methods of energy pumping are considered:

local input if the pumping region is much narrower than the relaxation zone. In this case, the pumping region can be considered as a surface on which the vibrational temperature changes in a jumplike manner;

extended input if the pumping region has a finite width. The energy introduced into the flow is defined as a function $I_{pump} = I_0 \exp(-(x - x_0)^2/d)$, where the parameter d was chosen to be equal approximately to one third of the relaxation-zone length.

Such a formulation of the problem corresponds to real conditions and allows one to estimate the effect of the pumping-region length. It is well known that part of the energy during pumping is spent on direct heating of the gas (bypassing vibrational degrees of freedom). It is possible to include this mechanism within the framework of approximations considered by introducing a source term into the energy equation for translational degrees of freedom. Nevertheless, the real kinetic scheme of such heating is rather complicated, and such a consideration is inexpedient within the framework of a simple kinetic model used.

Energy pumping leads to separation of vibrational and translational temperatures, though later these temperatures start leveling out owing to VT relaxation.

We consider interaction of small hydrodynamic perturbations (acoustic, thermal, and vortex ones) and calculate the transmission and reflection coefficients for all types of perturbations being generated.

General Algorithm of Problem Solution. The initial system of hydrodynamic equations with allowance for energy release and relaxation has the form

$$\begin{gathered}
\frac{d\rho}{dt} + \rho \operatorname{div} v = 0, \\
\rho \frac{dv}{dt} = -\operatorname{grad} p, \\
\frac{\gamma}{\gamma - 1} \frac{dT}{dt} - \frac{T}{\rho} \frac{d\rho}{dt} = \frac{m}{k_{\mathrm{B}}} \frac{\varepsilon - \varepsilon_{eq}}{\tau}, \\
\frac{d\varepsilon}{dt} = -\frac{\varepsilon - \varepsilon_{eq}}{\tau} + I_{pump},
\end{gathered} \tag{1}$$

where p, ρ, T, and v are the pressure, density, translational temperature, and velocity of the gas, respectively, γ is the ratio of specific heats, m is the mass of the molecule, I_{pump} is the power of energy pumping, k_{B} is the Boltzmann constant, τ is the relaxation time, h is the Planck constant, ω is the frequency, $\varepsilon(T_V) = (h\omega/m)/(\exp(h\omega/(k_{\mathrm{B}}T_V)) - 1)$ is the current vibrational energy of the gas, T_V is the vibrational temperature, and $\varepsilon_{eq}(T)$ is the equilibrium vibrational temperature.

The kinetic scheme used is as simple as possible and includes only vibrational relaxation within the framework of a simple relaxation equation [1] and pumping into vibrational degrees of freedom. The reason for these simplifications is the fact that, from the viewpoint of hydrodynamic perturbations, the main role belongs to energy-consuming processes with energy release into translational degrees of freedom, because it is this type of energy release that changes the hydrodynamic parameters. At the same time, such a simple formulation makes it possible to identify the basic mechanisms of amplification of perturbations and the instability region, which can be further refined as the model used becomes more complicated.

The solution of system (1) in a steady one-dimensional case yields the profiles of unperturbed parameters.

Interaction of small hydrodynamic perturbations with the nonequilibrium region is described by the solution of a linearized system for perturbations of the form $a'(x,y) = a'_0(x)\exp(i\omega t + ik_y y)$ (for the one-dimensional case, $k_y = 0$):

$$i\omega\rho'_0 + \rho'_0\,\frac{\partial v}{\partial x} + v\,\frac{\partial\rho'_0}{\partial x} + v'_{0x}\,\frac{\partial\rho}{\partial x} + \rho\,\frac{\partial v'_{0x}}{\partial x} + ik_y\rho v'_{0y} = 0,$$

$$i\omega\rho v'_{0x} + \rho v\,\frac{\partial v'_{0x}}{\partial x} + (\rho v'_{0x} + \rho'_0 v)\,\frac{\partial v}{\partial x} + \rho\,\frac{\partial T'_0}{\partial x} + T\,\frac{\partial\rho'_0}{\partial x} + T'_0\,\frac{\partial\rho}{\partial x} + \rho'_0\,\frac{\partial T}{\partial x} = 0,$$

$$i\omega\rho v'_{0y} + \rho v\,\frac{\partial v'_{0y}}{\partial x} + ik_y\rho T'_0 + ik_y\rho'_0 T = 0, \qquad (2)$$

$$\frac{\gamma}{\gamma-1}\left(i\omega T'_0 + v'_{0x}\,\frac{\partial T}{\partial x} + v\,\frac{\partial T'_0}{\partial x}\right) - \left(\frac{T'_0}{\rho} + \frac{T\rho'_0}{\rho^2}\right)v\,\frac{\partial\rho}{\partial x} - \frac{T}{\rho}\left(i\omega\rho'_0 + v\,\frac{\partial\rho'_0}{\partial x} + v'_{0x}\,\frac{\partial\rho}{\partial x}\right) =$$

$$= \frac{\varepsilon'_0}{\tau} - \frac{\varepsilon - \varepsilon_{eq}}{\tau^2}\,\frac{d\tau}{dT}\,T'_0 - \frac{\varepsilon'_{eq0}}{\tau},$$

$$i\omega\varepsilon'_0 + v\,\frac{\partial\varepsilon'_0}{\partial x} + v'_{0x}\,\frac{\partial\varepsilon}{\partial x} = -\frac{\varepsilon'_0}{\tau} + \frac{\varepsilon - \varepsilon_{eq}}{\tau^2}\,\frac{d\tau}{dT}\,T'_0 + \frac{\varepsilon'_{eq0}}{\tau},$$

System (2) is a system of linear equations. This means that the solution can be expanded in a certain set of functions. This statement is valid both in the nonequilibrium region and in the equilibrium regions (cold equilibrium region ahead of the relaxation zone and heated equilibrium region formed when the relaxation is completed). In equilibrium regions, however, system (2) becomes linear and has constant coefficients; the set of eigenfunctions (modes) in this case is well known [2]: two acoustic, thermal, vortex, and relaxation modes. We consider perturbations that can be generated by the relaxation zone. They are acoustic, vortex, thermal, and relaxation modes in the heated gas; in the cold gas, it is only an acoustic wave propagating upstream (we consider a subsonic flow only).

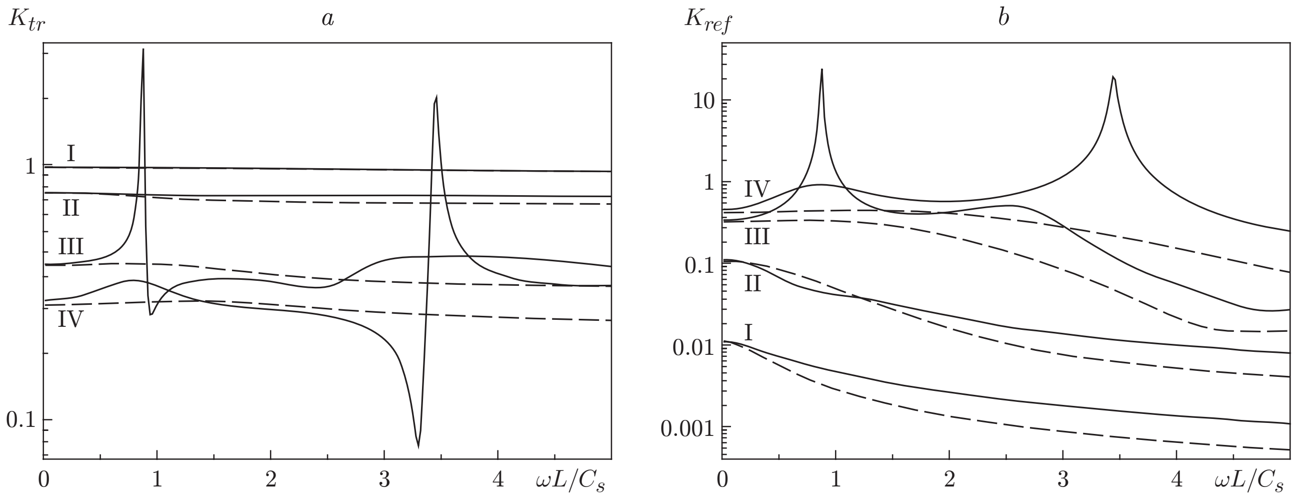


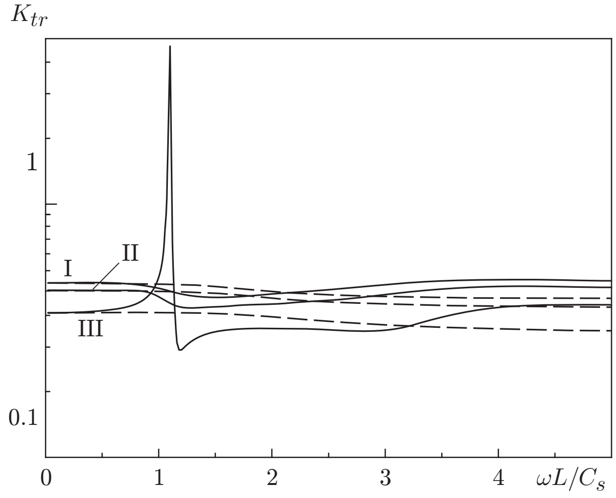
Fig. 1. Transmission coefficient K_{tr} (a) and reflection coefficient K_{ref} (b) versus the dimensionless frequency $\omega L/C_S$ in the case of local pumping of energy for v = 100 m/sec: $T_V = 500$ K (I), $T_V = 1000$ K (II), $T_V = 2000$ K (III), and $T_V = 2500$ K (IV); the dashed curves show the results calculated with ignored pumping of energy into translational degrees of freedom.

Division into modes persists in the relaxation zone as well, but the structure of modes is more complicated here.

The following algorithm is used. Relations for each mode propagating in the flow are used as the initial conditions in the equilibrium heated region. Integrating each model separately over the nonequilibrium region from right to left and matching with free-stream modes on the relaxation-zone front, we obtain a matrix consisting of perturbation amplitudes with dimension 4×4 for the one-dimensional case and 5×5 for the two-dimensional case.

If we define an acoustic, thermal, or vortex wave incident onto the nonequilibrium region, we can calculate the transmission and reflection coefficients corresponding to each mode, i.e., obtain the characteristics of the nonequilibrium zone as a resonator. Instability will arise in the case of an infinitely large transmission coefficient, i.e., in a situation when the nonequilibrium region itself generates perturbations.

One-Dimensional Interaction in the Flow. Interaction of the incident acoustic wave with the



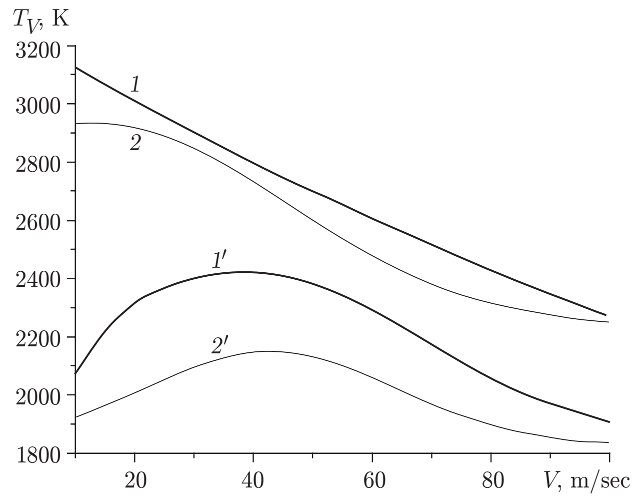


Fig. 2

Fig. 3

Fig. 2. Transmission coefficient K_{tr} versus the dimensionless frequency $\omega L/C_S$ in the case of extended pumping of energy for $v = 100$ m/sec: $T_V = 1500$ K (I), $T_V = 1602$ K (II), and $T_V = 1900$ K (III); the dashed curves show the results calculated with ignored pumping of energy into translational degrees of freedom.

Fig. 3. Boundaries of resonant interaction (above the critical curves $K_{tr} > 3$): curves 1 and $1'$ refer to the one-dimensional case, and curves 2 and $2'$ refer to the two-dimensional case; curves 1 and 2 refer to the local input of energy, and curves $1'$ and $2'$ refer to the extended input of energy.

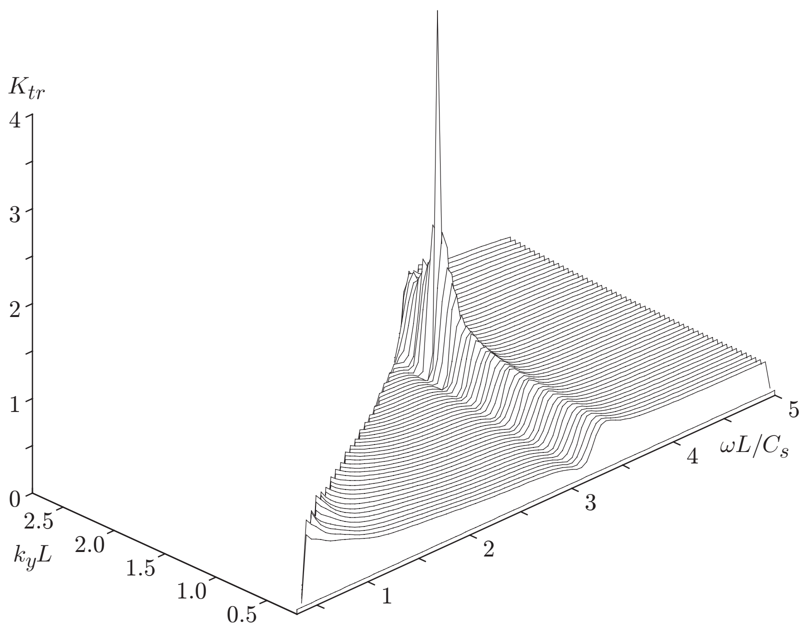


Fig. 4. Transmission coefficient K_{tr} versus the dimensionless frequency $\omega L/C_S$ and wave vector k_yL in the case of the local input of energy for $v = 100$ m/sec and

nonequilibrium region is described by transmission and reflection coefficients of the acoustic wave, which are determined by the ratio of amplitudes of generated perturbations to the amplitude of the incident wave. The interaction process includes two mechanisms. The first one is related to reflection of the incident acoustic wave from the region with gradients of hydrodynamic parameters; the second mechanism involves the transition of energy from internal degrees of freedom to translational ones.Figure 1 shows the calculated transmission and reflection coefficients. The calculations were performed for oxygen with the relaxation time $p\tau = 1.14 \cdot 10^{-10} \exp{(19.57(T/T_0)^{-1/3})}$ atm · sec [1]. If the vibrational temperature T_V is not very high, the transmission coefficient (K_{tr}) is approximately equal to unity and the reflection coefficient (K_{ref}) is rather small, i.e., the incident acoustic wave passes through the nonequilibrium region almost without any changes. With increasing T_V, the gradients of hydrodynamic parameters increase, and the incident acoustic wave is reflected to a greater extent; therefore, K_{tr} decreases and K_{ref} increases. Beginning from a certain value of T_V, however, a drastic increase in transmission and reflection coefficients occurs at certain frequencies. If we eliminate energy pumping from internal to translational degrees of freedom in equations for perturbations, the drastic increase in the coefficients K_{tr} and K_{ref} is not observed (dashed curves in Fig. 1). Thus, feeding of perturbations owing to the energy of internal degrees of freedom plays the main role in formation of anomalies in the behavior of K_{tr} and K_{ref}.

A similar consideration was performed for the case of extended pumping (Fig. 2). A comparison of Figs. 1a and 2 reveals the effect of the pumping zone length: the number of resonances is different, but the resonance is present in both models.

For each method of energy pumping and for a given flow velocity, there is a vibrational temperature T_V above which the growth rate coefficient will exceed a certain value at certain frequencies. Thus, it is possible to plot the dependence $T_V(v)$. In Fig. 3, $K_{tr} > 3$ lies higher than curves 1 and $1'$. Substantial pumping of energy from the nonequilibrium region to hydrodynamic perturbations occurs in these regions.

Two-Dimensional Interaction with the Nonequilibrium Region. To consider interactions with vortex perturbations and in the case of oblique incidence of acoustic waves, one has to solve a two-dimensional problem.

The solution is completely similar to the one-dimensional case. In the two-dimensional case, we have one more equation of motion for the velocity component v'_y, and an arbitrary perturbation is written in the form $a'(x, y) = a'_0(x) \exp(i\omega t + ik_y y)$. Figure 4 shows the transmission coefficient K_{tr} as a function of the dimensionless parameters $\omega L/C_S$ and $k_y L$ for fixed values of T_V and v, where $L = v_0\tau_0$, i.e., K_{tr} is determined by the gas parameters at the beginning of the relaxation region. It is seen that the growth-rate coefficients in the resonant zone are significantly higher in the two-dimensional problem, which is related to an increase in the path in an amplifying medium for oblique perturbations. The calculations were performed for the case with a reflected acoustic wave. As in the one-dimensional case, the boundary of resonant interaction can be found (see Fig. 3). In the two-dimensional case, the boundary is lower than that in the one-dimensional case, i.e., the region of resonant interaction becomes larger.

Effect of Resonator Walls on the Properties of the Gas-Flow System. A real system contains walls and mirrors deflecting the flow. Vortex and thermal perturbations are entrained together with the flow, whereas acoustic perturbations are reflected from the walls. The simplest estimates show that such obstacles substantially increase the transmission and reflection coefficients.

Interaction of Vortex Perturbations with the Relaxation Zone. This problem is of interest because of attempts to improve heat removal in the system by means of flow turbulization.

Additional turbulization of a gas-dynamic gas flow is assumed to improve stability of the discharge. Enhancing mixing in the working region, it should reduce the arising inhomogeneities in hydrodynamic parameters, which yields a more stable discharge.

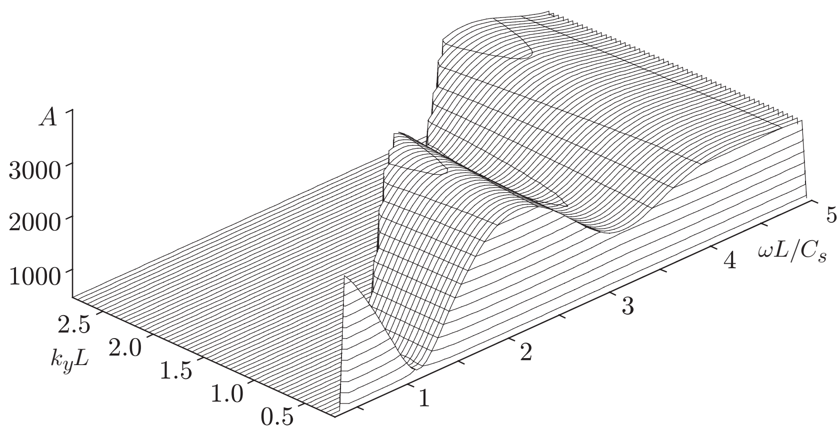


Fig. 5. Efficiency of generation of the thermal mode versus the dimensionless frequency $\omega L/C_S$ and wave vector k_yL in the case of the local pumping of energy for $v = 50$ m/sec and $T_V = 2000$ K.

It was shown in [3–6], however, that stability decreases with increasing turbulence intensity. Within the framework of the problem considered, this phenomenon can be attributed to generation of heat waves.

Turbulization of the flow within the framework of the problem solved is equivalent to origination of a vortex mode incident onto the perturbation front. The technique used in the present work makes it possible to demonstrate that the vortex mode incident onto the nonequilibrium region leads to generation of the entire range of hydrodynamic modes, including the thermal mode, which exerts the greatest effect on discharge destabilization.

Figure 5 shows the ratio of the amplitude of the generated thermal mode to the amplitude of the incident vortex mode as a function of $\omega L/C_S$ and k_yL. There is a range of values of $\omega L/C_S$ and k_yL in which the efficiency of generation of the thermal mode is much lower than for other wavenumbers. Thus, it is possible to choose a turbulization mode providing the minimum generation of thermal perturbations.

Generation of heat waves may be responsible for the nonmonotonic dependence of the limiting input power on the degree of nonequilibrium and pumping velocity, which was observed in experiments with turbulization. The amplitude of the heat waves is determined by parameters of vortex perturbations and can be minimized, which would favor discharge stabilization.

Conclusions. The problem of interaction of small hydrodynamic perturbations with a nonequilibrium region of a gas flow with different methods of energy pumping is solved.

In the case of one-dimensional interaction, there exists a range of resonant parameters determined by the pumping velocity, degree of nonequilibrium, and perturbation frequency for which the transmission and reflection coefficients are anomalously high. It is shown that additional obstacles substantially increase the values of the resonant coefficients.

The problem of two-dimensional interaction of perturbations with a nonequilibrium gas region is solved. It is found that the resonant transmission and reflection coefficients are higher than those in the one-dimensional case.

It is also shown that interaction of vortex perturbations with the nonequilibrium region generates heat waves whose amplitude is determined by flow parameters.

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